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LETTER TO THE EDITOR

Integrable Kondo impurities in the one-dimensional supersymmetric U model of strongly correlated electrons

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Abstract. Integrable Kondo impurities in the one-dimensional supersymmetric U model of strongly correlated electrons are studied by means of the boundary graded quantum inverse scattering method. The boundary K-matrices depending on the local magnetic moments of the impurities are presented as non-trivial realizations of the reflection equation algebras in an impurity Hilbert space. Furthermore, the model Hamiltonian is diagonalized and the Bethe ansatz equations are derived. It is interesting to note that our model exhibits a free parameter in the bulk Hamiltonian but no free parameter exists on the boundaries. This is in sharp contrast to the impurity models arising from the supersymmetric t–J and extended Hubbard models where there is no free parameter in the bulk but there is a free parameter on each boundary.

Recently, there has been substantial research devoted to the investigation of the theory of impurities coupled to Luttinger liquids. Such a problem was first considered by Lee and Toner [1]. By using the perturbative renormalization group theory they found that the Kondo temperature crosses from a generic power-law dependence on the Kondo coupling constant to an exponential one in the infinite limit. Afterwards, a 'poor man's' scaling procedure was carried out by Furusaki and Nagaosa [2] ,who found a stable strong-coupling fixed point for both antiferromagnetic and ferromagnetic cases. On the other hand, boundary conformal field theory, first developed by Affleck and Ludwig [3] for the conventional Kondo problem based on a previous work by Nozières [4], leads us to a classification of critical behaviour for the Kondo problem in the presence of the electron–electron interactions [5]. It turns out that there are two types of critical behaviour, i.e., either a local Fermi liquid with standard low-temperature thermodynamics or the non-Fermi liquid observed by Furusaki and Nagaosa [2]. However, in order to get a full picture about the critical behaviour of Kondo impurities coupled to Luttinger liquids, some simple integrable models, as in the conventional Kondo problem which allow exact solutions [6, 7], are desirable.

Several integrable magnetic or nonmagnetic impurity problems describing impurities embedded in systems of correlated electrons have so far appeared in the literature. Among them are versions of the supersymmetric t–J model with impurities [8–10]. Such an idea to incorporate an impurity into a closed chain dates back to Andrei and Johannesson [11] (see also [12, 13]). However, the model thus constructed suffers from a lack of backward scattering and results in a very complicated Hamiltonian which is diffficult to justify on physical grounds. Therefore, as observed by Kane and Fisher [14], it is advantageous to adopt open boundary

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conditions with the impurities situated at the ends of the chain when studying Kondo impurities coupled to integrable strongly correlated electron systems [15, 16].

In this letter, we study integrable Kondo impurities in the one-dimensional supersymmetric U model of strongly correlated electrons, which has been extensively studied in [17–20]. Two different non-c-number boundary K-matrices are constructed, which turn out to be quite different from those for the t–J and the supersymmetric extended Hubbard models [16, 21], due to the fact that no free parameter exists. However, it should be emphasized that our new non-c-number boundary K-matrices are highly non-trivial, in the sense that they cannot be factorized into the product of a c-number boundary K-matrix and the corresponding local monodromy matrices. Integrability of the models is established by relating the Hamiltonians to one-parameter families of commuting transfer matrices. The model is solved by means of the coordinate Bethe ansatz method and Bethe ansatz equations are derived. It is interesting to note that our model exhibits a free parameter in the bulk Hamiltonian but no free parameter exists on the boundaries. This is in sharp contrast to the impurity models arising from the supersymmetric t–J and extended Hubbard models where there is no free parameter in the bulk but there is a free parameter on each boundary.

Let $c^\dagger_{j,\sigma}$ and $c_{j,\sigma}$ denote the creation and annihilation operators of the conduction electrons with spin σ at site j, which satisfy the anti-commutation relations given by $\{c^\dagger_{i,\sigma}, c_{j,\tau}\} = \delta_{ij}\delta_{\sigma\tau}$, where $i, j = 1, 2, \ldots, L$ and $\sigma, \tau = \uparrow, \downarrow$. Consider the Hamiltonian which describes two impurities coupled to the supersymmetric U open chain

$$\begin{aligned}
H = &-\sum_{j=1}^{L-1}\sum_{\sigma}(c^\dagger_{j\sigma}c_{j+1\sigma} + \text{h.c.})\exp(-\tfrac{1}{2}\eta n_{j,-\sigma} - \tfrac{1}{2}\eta n_{j+1,-\sigma}) \\
&+t_p\sum_{j=1}^{L-1}(c^\dagger_{j\uparrow}c^\dagger_{j\downarrow}c_{j+1\downarrow}c_{j+1\uparrow} + \text{h.c.}) + U\sum_{j=1}^{L} n_{j\uparrow}n_{j\downarrow} \\
&+J_a\boldsymbol{S}_a\cdot\sum_{\sigma,\sigma'}\tau_{\sigma\sigma'}c^\dagger_{1\sigma}c_{1\sigma'} + V_a n_1 + U_a n_{1\uparrow}n_{1\downarrow} \\
&+J_b\boldsymbol{S}_b\cdot\sum_{\sigma,\sigma'}\tau_{\sigma\sigma'}c^\dagger_{L\sigma}c_{L\sigma'} + V_b n_L + U_b n_{L\uparrow}n_{L\downarrow}
\end{aligned}\tag{1}$$

where J_g, V_g and U_g $(g = a, b)$ are, respectively, the Kondo coupling constants, the impurity scalar potentials and the boundary Hubbard-like interaction constants; $\boldsymbol{\tau} \equiv (\tau_x, \tau_y, \tau_z)$ are the usual Pauli matrices with indices $|1\rangle = |\downarrow\rangle$ and $|2\rangle = |\uparrow\rangle$; $\boldsymbol{S}_g(g = a, b)$ are the local moments with spin-$\frac{1}{2}$ located at the left and right ends of the system respectively, and $t_p = \frac{U}{2} = \mathrm{e}^{-\eta} - 1$; $n_{j\sigma}$ is the number density operator $n_{j\sigma} = c^\dagger_{j\sigma}c_{j\sigma}$, $n_j = n_{j\uparrow} + n_{j\downarrow}$. Below we will establish the quantum integrability of the model (1) for the following four choices of the coupling constants: case A: $J_g = 2\alpha + 2$, $V_g = (\alpha - 1)/2$, $U_g = -(\alpha^2 + \alpha + 1)/\alpha$; case B: $J_g = -4(\alpha + 1)/[(2\alpha - 1)(2\alpha + 3)]$, $V_g = 3/[(2\alpha - 1)(2\alpha + 3)]$, $U_g = 3/[\alpha(2\alpha - 1)(2\alpha + 3)]$; case C: $J_a = 2\alpha + 2$, $V_a = (\alpha - 1)/2$, $U_a = -(\alpha^2 + \alpha + 1)/\alpha$, $J_b = -4(\alpha + 1)/[(2\alpha - 1)(2\alpha + 3)]$, $V_b = 3/[(2\alpha - 1)(2\alpha + 3)]$, $U_b = 3/[\alpha(2\alpha - 1)(2\alpha + 3)]$; case D: $J_a = -4(\alpha + 1)/[(2\alpha - 1)(2\alpha + 3)]$, $V_a = 3/[(2\alpha - 1)(2\alpha + 3)]$, $U_a = 3/[\alpha(2\alpha - 1)(2\alpha + 3)]$, $J_b = 2\alpha + 2$, $V_b = (\alpha - 1)/2$, $U_b = -(\alpha^2 + \alpha + 1)/\alpha$. Here and hereafter, $\alpha = 2/U$. This is achieved by showing that the Hamiltonian can be derived from the graded boundary quantum inverse scattering method. Indeed, the Hamiltonian of the supersymmetric U model with the periodic boundary conditions commutes with the transfer matrix, which is the supertrace of the monodromy matrix $T(u)$,

$$T(u) = R_{0L}(u)\ldots R_{01}(u).\tag{2}$$

The explicit form of the quantum R-matrix $R_{0j}(u)$ is given in [17]. Here u is the spectral parameter, and the subscript 0 denotes the auxiliary superspace $V = \boldsymbol{C}^{2,2}$. It should be noted that the supertrace is carried out for the auxiliary superspace V. The elements of the supermatrix $T(u)$ are the generators of an associative superalgebra $\mathcal{A}$ defined by the relations

$$R_{12}(u_1 - u_2) \overset{1}{T}(u_1) \overset{2}{T}(u_2) = \overset{2}{T}(u_2) \overset{1}{T}(u_1) R_{12}(u_1 - u_2) \tag{3}$$

where $\overset{1}{X} \equiv X \otimes 1$, $\overset{2}{X} \equiv 1 \otimes X$ for any supermatrix $X \in \mathrm{End}(V)$. For later use, we list some useful properties enjoyed by the R-matrix: (i) unitarity: $R_{12}(u)R_{21}(-u) = 1$ and (ii) crossing-unitarity: $R_{12}^{st_2}(-u+2)R_{21}^{st_2}(u) = \tilde{\rho}(u)$ with $\tilde{\rho}(u)$ being a scalar function, $\tilde{\rho}(u) = u^2(2-u)^2/[(2+2\alpha-u)^2(2\alpha+u)^2]$.

In order to describe integrable electronic models on open chains, we introduce two associative superalgebras $\mathcal{T}_-$ and $\mathcal{T}_+$ defined by the R-matrix $R(u_1 - u_2)$ and the relations

$$\begin{aligned} &R_{12}(u_1 - u_2) \overset{1}{\mathcal{T}}_-(u_1) R_{21}(u_1 + u_2) \overset{2}{\mathcal{T}}_-(u_2) \\ &\qquad = \overset{2}{\mathcal{T}}_-(u_2) R_{12}(u_1 + u_2) \overset{1}{\mathcal{T}}_-(u_1) R_{21}(u_1 - u_2) \end{aligned} \tag{4}$$

$$\begin{aligned} &R_{21}^{st_1 ist_2}(-u_1 + u_2) \overset{1}{\mathcal{T}}{}_+^{st_1}(u_1) R_{12}(-u_1 - u_2 + 2) \overset{2}{\mathcal{T}}{}_+^{ist_2}(u_2) \\ &\qquad = \overset{2}{\mathcal{T}}{}_+^{ist_2}(u_2) R_{21}(-u_1 - u_2 + 2) \overset{1}{\mathcal{T}}{}_+^{st_1}(u_1) R_{12}^{st_1 ist_2}(-u_1 + u_2) \end{aligned} \tag{5}$$

respectively. Here the supertransposition st_μ $(\mu = 1, 2)$ is only carried out in the μth componant of the superspace $V \otimes V$, whereas ist_μ denotes the inverse operation of st_μ. By modifying Sklyanin's arguments [22], one may show that the quantities $\tau(u)$ given by $\tau(u) = str(\mathcal{T}_+(u)\mathcal{T}_-(u))$ constitute a commutative family, i.e. $[\tau(u_1), \tau(u_2)] = 0$ [23, 24].

One can obtain a class of realizations of the superalgebras $\mathcal{T}_+$ and $\mathcal{T}_-$ by choosing $\mathcal{T}_\pm(u)$ to be of the form

$$\mathcal{T}_-(u) = T_-(u)\tilde{\mathcal{T}}_-(u)T_-^{-1}(-u) \qquad \mathcal{T}_+^{st}(u) = T_+^{st}(u)\tilde{\mathcal{T}}_+^{st}(u)(T_+^{-1}(-u))^{st} \tag{6}$$

with

$$T_-(u) = R_{0M}(u)\dots R_{01}(u) \qquad T_+(u) = R_{0L}(u)\dots R_{0,M+1}(u) \qquad \tilde{\mathcal{T}}_\pm(u) = K_\pm(u) \tag{7}$$

where $K_\pm(u)$, called boundary K-matrices, are representations of $\mathcal{T}_\pm$ in some representation superspace. Although many attempts have been made to find c-number boundary K-matrices, which may be referred to as the fundamental representation, it is no doubt very interesting to search for non-c-number K-matrices, arising as representations in some Hilbert spaces, which may be interpreted as impurity Hilbert spaces [16].

We now solve (4) and (5) for $K_-(u)$ and $K_+(u)$. Quite interestingly, for the supersymmetric U model [17], there are two different non-c-number boundary K-matrices. One is

$$K_-^I(u) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & A_-^I(u) & B_-^I(u) & 0 \\ 0 & C_-^I(u) & D_-^I(u) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{8}$$

where $A_-^I(u) = -(u^2 - 2u + 4 - u\boldsymbol{S}_a^z)/Z_-^I$, $B_-^I(u) = 2u\boldsymbol{S}_a^-/Z_-^I$, $C_-^I(u) = 2u\boldsymbol{S}_a^+/Z_-^I$, $D_-^I(u) = -(u^2 - 2u + 4 + u\boldsymbol{S}_a^z)/Z_-^I$, $Z_-^I \equiv (u-2)(u+2)$, and the other takes the form,

$$K_-^{II}(u) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & A_-^{II}(u) & B_-^{II}(u) & 0 \\ 0 & C_-^{II}(u) & D_-^{II}(u) & 0 \\ 0 & 0 & 0 & F_-(u) \end{pmatrix} \tag{9}$$

with $A_-^{II}(u) = -(u^2 - 2u - 4\alpha^2 - 4\alpha + 3 - u\boldsymbol{S}_a^z)/Z_-^{II}$, $B_-^{II}(u) = 2u\boldsymbol{S}_a^-/Z_-^{II}$, $C_-^{II}(u) = 2u\boldsymbol{S}_a^+/Z_-^{II}$, $D_-^{II}(u) = -(u^2 - 2u - 4\alpha^2 - 4\alpha + 3 + u\boldsymbol{S}_a^z)/Z_-^{II}$, $F_-(u) = ((u + 2\alpha - 1)(u + 2\alpha + 3))/Z_-^{II}$, $Z_-^{II} \equiv (u - 2\alpha + 1)(u - 2\alpha - 3)$. Here $\boldsymbol{S}^\pm = \boldsymbol{S}^x \pm \mathrm{i}\boldsymbol{S}^y$. The matrix $K_+(u)$ can be obtained from the isomorphism of the superalgebras $\mathcal{T}_-$ and $\mathcal{T}_+$. Indeed, given a solution $K_-(u)$ of the equation (4), then $K_+(u)$ defined by $K_+^{st}(u) = K_-(-u + \frac{1}{2})$ is a solution of the equation (5). The proof follows from some algebraic computations by making use of the properties of the R-matrix [24]. Therefore, one may choose the boundary matrix $K_+(u)$ as

$$K_+^I(u) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & A_+^I(u) & B_+^I(u) & 0 \\ 0 & C_+^I(u) & D_+^I(u) & 0 \\ 0 & 0 & 0 & F_+(u) \end{pmatrix} \tag{10}$$

where $A_+^I(u) = -(u^2 - 4\alpha^2 - 4\alpha + 2 - (u-1)\boldsymbol{S}_b^z)/Z_+^I$, $B_+^I(u) = 2(u-1)\boldsymbol{S}_b^-/Z_+^I$, $C_+^I(u) = 2(u-1)\boldsymbol{S}_b^+/Z_+^I$, $D_+^I(u) = -(u^2 - 4\alpha^2 - 4\alpha + 2 + (u-1)\boldsymbol{S}_b^z)/Z_+^I$, $F_+(u) = ((u - 2\alpha)(u - 2\alpha - 4))/Z_+^I$, $Z_+^I \equiv (u + 2\alpha + 2)(u + 2\alpha - 2)$, and

$$K_+^{II}(u) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & A_+^{II}(u) & B_+^{II}(u) & 0 \\ 0 & C_+^{II}(u) & D_+^{II}(u) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{11}$$

where $A_+^{II}(u) = -(u^2 + 3 - (u-1)\boldsymbol{S}_b^z)/Z_+^{II}$, $B_+^{II}(u) = 2(u-1)\boldsymbol{S}_b^-/Z_+^{II}$, $C_+^{II}(u) = 2(u-1)\boldsymbol{S}_b^+/Z_+^{II}$, $D_+^{II}(u) = -(u^2 + 3 + (u-1)\boldsymbol{S}_b^z)/Z_+^{II}$, $Z_+^{II} \equiv (u+1)(u-3)$.

As usual, the boundary transfer matrix $\tau(u)$ may be rewritten as

$$\tau(u) = \mathrm{str}[K_+(u)T(u)K_-(u)T^{-1}(-u)]. \tag{12}$$

Since $K_\pm(u)$ can be taken as $K_\pm^I(u)$ or $K_\pm^{II}(u)$, respectively, we have four possible choices of boundary transfer matrices, which reflect the fact that the boundary conditions on the left end and on the right end of the open lattice chain are independent. Then it can be shown [23,24] that Hamiltonians corresponding to all four choices can be embedded into the above four boundary transfer matrices, respectively. Indeed, the Hamiltonian (1) is related to the transfer matrix $\tau(u)$ (up to an unimportant additive chemical potential term)

$$\begin{aligned} H^R &\equiv -\frac{U}{2(U+2)}H = \frac{\tau''(0)}{4(V+2W)} \\ &= \sum_{j=1}^{L-1} H_{j,j+1}^R + \frac{1}{2}\overset{1}{K}{}'_-(0) + \frac{1}{2(V+2W)}\Bigg[\mathrm{str}_0\left(\overset{0}{K}_+(0)G_{L0}\right) \\ &\quad + 2\,\mathrm{str}_0\left(\overset{0}{K}{}'_+(0)H_{L0}^R\right) + \mathrm{str}_0\left(\overset{0}{K}_+(0)\left(H_{L0}^R\right)^2\right)\Bigg] \end{aligned} \tag{13}$$

where $V = \mathrm{str}_0\, K_+'(0)$, $W = \mathrm{str}_0(\overset{0}{K}_+(0)H_{L0}^R)$, $H_{i,j}^R = P_{i,j}R_{i,j}'(0)$, $G_{i,j} = P_{i,j}R_{i,j}''(0)$, with $P_{i,j}$ being the graded permutation operator acting on the ith and jth quantum spaces. (13) implies that the boundary supersymmetric U model admits an infinite number of conserved currents which are in involution with each other, thus assuring the integrability. It should be emphasized that Hamiltonian (1) appears as the second derivative of the transfer matrix $\tau(u)$ with respect to the spectral parameter u at $u = 0$. This is due to the fact that the supertrace of $K_+(0)$ equals zero. As pointed out in [24], the reason for the zero supertrace of $K_+(0)$ is related to the fact that the quantum space is a four-dimensional *typical* irreducible representation of $gl(2|1)$. A similar situation also occurs in the Hubbard-like models [25].

The Hamiltonian (1) may be diagonalized by means of the coordinate Bethe ansatz method. The Bethe ansatz equations are

$$\left(\frac{\theta_j - \frac{\mathrm{i}}{2}}{\theta_j + \frac{\mathrm{i}}{2}}\right)^{2L} \prod_{g=a,b} \frac{\theta_j - \theta_g + \mathrm{i}c}{\theta_j + \theta_g - \mathrm{i}c} = \prod_{\beta=1}^{M} \frac{\theta_j - \lambda_\beta + \mathrm{i}\frac{c}{2}}{\theta_j - \lambda_\beta - \mathrm{i}\frac{c}{2}} \cdot \frac{\theta_j + \lambda_\beta + \mathrm{i}\frac{c}{2}}{\theta_j + \lambda_\beta - \mathrm{i}\frac{c}{2}}$$

$$\prod_{g=a,b} \frac{(\lambda_\alpha + \frac{\mathrm{i}c}{2})^2 - \theta_g{}^2}{(\lambda_\alpha - \frac{\mathrm{i}c}{2})^2 - \theta_g{}^2} \prod_{j=1}^{N} \frac{\lambda_\alpha - \theta_j + \mathrm{i}\frac{c}{2}}{\lambda_\alpha - \theta_j - \mathrm{i}\frac{c}{2}} \cdot \frac{\lambda_\alpha + \theta_j + \mathrm{i}\frac{c}{2}}{\lambda_\alpha + \theta_j - \mathrm{i}\frac{c}{2}} \qquad (14)$$

$$= \prod_{\substack{\beta=1 \\ \beta\neq\alpha}}^{M} \frac{\lambda_\alpha - \lambda_\beta + \mathrm{i}c}{\lambda_\alpha - \lambda_\beta - \mathrm{i}c} \cdot \frac{\lambda_\alpha + \lambda_\beta + \mathrm{i}c}{\lambda_\alpha + \lambda_\beta - \mathrm{i}c}$$

where $c = \mathrm{e}^\eta - 1$, the charge rapidities $\theta_j \equiv \theta(k_j)$ are related to the single-particle quasi-momenta k_j by $\theta(k) = \frac{1}{2}\tan(\frac{k}{2})$ [18], and $\theta_a,\ \theta_b$ take the following form for the four choices: case A: $\theta_a = -\frac{\mathrm{i}}{2}$, $\theta_b = -\frac{\mathrm{i}}{2}$; case B: $\theta_a = \frac{\mathrm{i}}{U+2}$, $\theta_b = \frac{\mathrm{i}}{U+2}$; case C: $\theta_a = -\frac{\mathrm{i}}{2}$, $\theta_b = \frac{\mathrm{i}}{U+2}$; case D: $\theta_a = \frac{\mathrm{i}}{U+2}$, $\theta_b = -\frac{\mathrm{i}}{2}$. The corresponding energy eigenvalue E of the model is given by $E = -2\sum_{j=1}^{N}\cos k_j$, where we have dropped an additive constant.

In conclusion, we have studied integrable Kondo impurities coupled with the one-dimensional supersymmetric U open chain. The quantum integrability follows from the fact that the model Hamiltonian may be embbeded into a one-parameter family of commuting transfer matrices. Moreover, the Bethe ansatz equations are derived by means of the coordinate Bethe ansatz approach. It is quite interesting to note that in the boundary K-matrices (8) and (9), no free parameter is available, in contrast to the t–J and the supersymmetric extended Hubbard models [16, 21]. Further, it is desirable to investigate the thermodynamic equilibrium properties of the model, based on the Bethe ansatz equations (14). The details are deferred to another publication.

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Note added in proof. After completion of this paper, we noticed a preprint from H Hrahm and N A Slavnov entitled *New solution to the reflection equation and the projecting method* (cond-mat/9810312), where the method of [16] is generalized in the context of projection. We are grateful to H Frahm for bringing this reference to our attention.

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